For all questions, answer choice "E. NOTA" mean "None of the Above" answers is correct. Unless otherwise stated, assume NO figures are drawn to scale and length measurements are given in units.

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1. Determine the area of the region bounded by $f(x) = 2x$ and $g(x) = x^2 - 3$.

A) $32/3$ C) $16/3$ E) NOTA
B) $20/3$ D) $8/3$

2. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume of the sphere increasing when the radius is 20 inches?

A) 160π C) 1600π E) NOTA
B) 320π D) 3200π

3. A sphere and a cube have the same surface area. What is the ratio of the volume of the sphere to the volume of the cube?

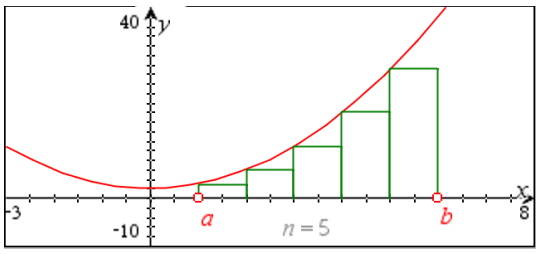
A) $\pi/2$ C) $\sqrt{3/\pi}$ E) NOTA
B) $4\pi/3$ D) $\sqrt{6/\pi}$

4. What is the area of the region in the plane between the graphs of $y = \sqrt{x}$ and $y = -2x$ between $x = 1$ and $x = 4$?

A) $31/3$ C) $59/3$ E) NOTA
B) $37/3$ D) $65/3$

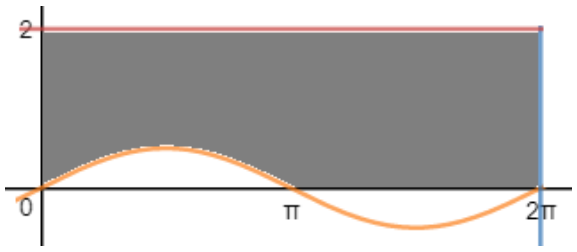
5. Given a graph of $f(x) = x^2 + 2$, find the left-endpoint Riemann sum using five uniform subintervals representing an approximation of $\int_1^6 x^2 + 2\, dx$.

A) 103
B) 65
C) 38
D) 27
E) NOTA

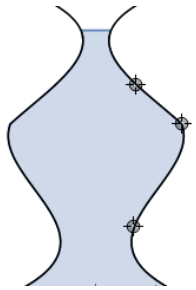


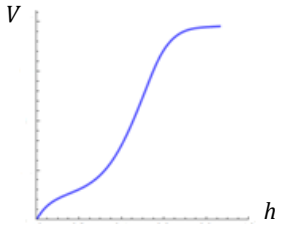
6. What is the area of the shaded region in the figure below, which is enclosed by the y-axis, $y = 2$, $x = 2\pi$, and the graph of $y = \frac{1}{2}\sin x$ that is above the x-axis ?

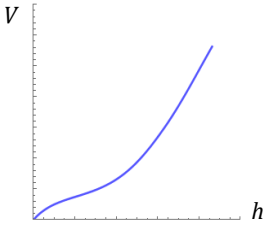
A) $4\pi - 2$
B) $4\pi - \frac{\sqrt{3}}{2}$
C) $4\pi - \frac{1}{2}$
D) $4\pi - 1$
E) NOTA

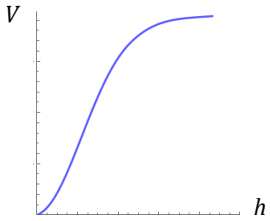


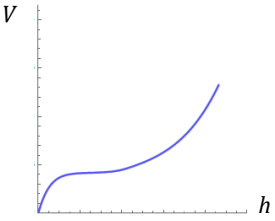
7. The graphic below represents the profile of a circularly symmetric container which was initially empty, then gradually filled with fluid. Which of the following graphs best represents the volume V of fluid versus the height h of the fluid?



A) 

B) 

C) 

D) 

E) NOTA

Graphics created with "Filling a Container Defined by a Curve" from the Wolfram Demonstrations Project http://demonstrations.wolfram.com/FillingAContainerDefinedByACurve/ Contributed by: Bruce Atwood (Beloit College)

8. For what radius r of a sphere will the rate of change of the volume with respect to the radius be numerically equal to the rate of change of the surface area with respect to the radius?

A) 1
B) 2
C) 3
D) 4
E) NOTA

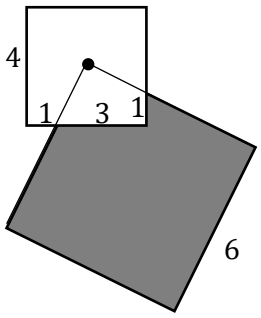
9. Let $f(x) = -(x-3)^2 + 2$ and $g(x) = x - 3$ on $[0, 4]$. Determine which of the following integrals would find the volume of a solid generated by revolving the area between the two curves about the line $y = -7$.

A) $V = \pi \int_0^1 (g(x) - 7)^2 - (f(x) - 7)^2 \, dx + \pi \int_1^4 (f(x) - 7)^2 - (g(x) - 7)^2 \, dx$
B) $V = \pi \int_0^1 (g(x) + 7)^2 - (f(x) + 7)^2 \, dx + \pi \int_1^4 (f(x) + 7)^2 - (g(x) + 7)^2 \, dx$
C) $V = \pi \int_0^1 [(g(x) + 7) - (f(x) + 7)]^2 \, dx + \pi \int_1^4 [(f(x) + 7) - (g(x) + 7)]^2 \, dx$
D) $V = \pi \int_0^1 [(g(x) - 7) - (f(x) - 7)]^2 \, dx + \pi \int_1^4 [(f(x) - 7) - (g(x) - 7)]^2 \, dx$
E) NOTA

10. The region enclosed by the graphs of $y = e^{x-2}$ and $y = -x$ between the vertical lines $x = 0$ and $x = 2$ is rotated about the line $y = -4$. Which of the following gives the volume of the generated solid?

A) $\pi \int_0^2 ((e^{x-2} - 4)^2 - (-x - 4)^2) dx$
B) $\pi \int_0^2 ((e^{x-2} + 4)^2 - (-x + 4)^2) dx$
C) $\pi \int_0^2 ((e^{x-2})^2 - (-x)^2 - 4^2) dx$
D) $\pi \int_{-2}^e ((\ln y - 2)^2 - (-y - 4)^2) dx$
E) NOTA

11. In the diagram at right, a square with side length 4 and a square with side length 6 overlap. One corner of the larger square is at the center of the smaller square. The sides of the square intersect, as shown. Find the area of the shaded region.



A) $36 - \sqrt{10}$
B) $36 - 2\sqrt{10}$
C) 30
D) 32
E) NOTA

12. Find the area enclosed by the graph of $9x^2 + 36y^2 - 36x + 288y + 288 = 0$.

A) 9π
B) 18π
C) 24π
D) 36π+15
E) 324

13. Find the area outside the graph of $|x| + |y| = 3$ and inside the graph of $x^2 + y^2 = 9$.

A) 9π
B) 9π - 9
C) 9π - 18
D) 36π+15
E) 324

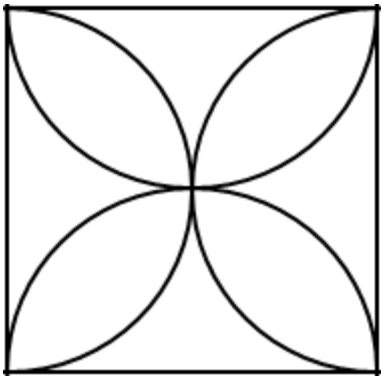
14. The vertex of the parabola $y^2 - 4y - 5x + 19 = 0$ and the two foci of $9y^2 - 16x^2 - 36y - 32x - 124 = 0$ are the vertices of a triangle. Find the area enclosed by this triangle.

A) 10
B) 20
C) 30
D) 40
E) 50

15. A pyramid has its base in the shape of a regular hexagon of side length 6. If the height of the pyramid is 15, find the volume of the pyramid.

A) 250
B) 270
C) $270\sqrt{3}$
D) $540\sqrt{3}$
E) $1620\sqrt{3}$

16. A 4-petal flower, formed with 4 semicircular arcs, is inscribed in a square of side length 12. What is the area enclosed by the flower?



A) $72(\pi - 2)$
B) $36(\pi - 2)$
C) $16(\pi - 2)$
D) $12(\pi - 2)$
E) NOTA

17. Find the volume for a solid of revolution found by rotating the region bounded by $y = x^3 + 1, x = 1,$ and $y = 1$ about the y-axis.

A) $8\pi/3$ C) $3\pi/7$ E) NOTA
B) $4\pi/13$ D) $2\pi/5$

18. Let the region bounded by $x^2 + y^2 = 9$ be the base of a solid. Find the volume of a solid of cross sections built by equilateral triangles perpendicular to the base from one edge of the circle to the other.

A) $36\sqrt{3}$ C) $72\sqrt{3}$ E) NOTA
B) $54\sqrt{3}$ D) $80\sqrt{3}$

19. What is the largest possible volume of a rectangular box whose diagonal length is 15?

A) $48^{3/2}$ C) $108^{3/2}$ E) NOTA
B) $75^{3/2}$ D) $120^{2/3}$

20. Find the area represented by the definite integral $\int_{-2}^{3} |x^2 - x - 2|\, dx$.

A) $-3/2$ C) $49/6$ E) NOTA
B) $31/6$ D) $53/6$

21. The area of the region bounded by the graphs of equations $y = 2x + 1$ and $y = x^2 - 2$ is a rational number which can be written in reducted form as $\frac{2^a}{3^b}$, for some integers a and b. What is $a(b + a)$?

A) 10 C) 25 E) NOTA
B) 15 D) 30

22. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 9 - x^2$.

A) $2\sqrt{3} \times 6$ C) $\sqrt{3} \times 8$ E) NOTA
B) $\sqrt{3} \times 5$ D) 3×5

23. The area inside the polar equation $r = 3\sin\theta$ and above the lines $y = x$ and $y = -x$ is given by

A) $9\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2\theta\, d\theta$

B) $9\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\theta\ d\theta$

C) $\frac{9}{2}\int_{\pi/4}^{3\pi/4} \sin^2\theta\, d\theta$

D) $\frac{3}{2}\int_{-1}^{1} \sin^2\theta\ d\theta$

E) NOTA

24. Water is poured into a conical cup at a rate of $\frac{3}{4}$ cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast is the water level rising when the water is 4 inches deep?

A) $\frac{3\pi}{8}\ in/_{sec}$ C) $\frac{4\pi}{3}\ in/_{sec}$ E) NOTA

B) $\frac{3\pi}{2}\ in/_{sec}$ D) $\frac{\pi}{4}\ in/_{sec}$

25. A rectangle has two sides along the positive coordinate axes and its upper right hand corner point lies on the curve, $x^3 - 2xy^2 + y^3 - 1 = 0$. How fast is the area of the rectangle changing, in square units per second, as the point passes through the position (1, 2) if it is moving so that $\frac{dx}{dt} = 3$ units per second?

A) 39/4 C) 43/4 E) NOTA

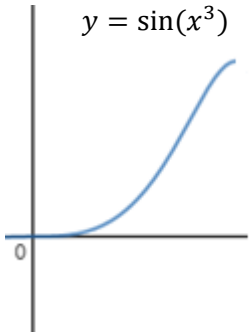
B) 63/8 D) 59/8

26. Let A=(-5, 5), B=(1, 3), and C=(5, 3) be three points in the plane, forming triangle ABC. Find the volume of the solid obtained by revolving the triangle around the line containing the altitude from point A.

A) 80π C) 48π E) NOTA

B) 64π D) 24π

27. Let $A(t)$ be the area under the curve, $y = \sin(x^3)$, $0 \le x \le t$. Let $B(t)$ be the area of the triangle with vertices at $(0, 0)$, $(t, 0)$, and $(t, \sin(t^3))$. Find $\lim_{t\to 0^+} \frac{B(t)}{A(t)}$.



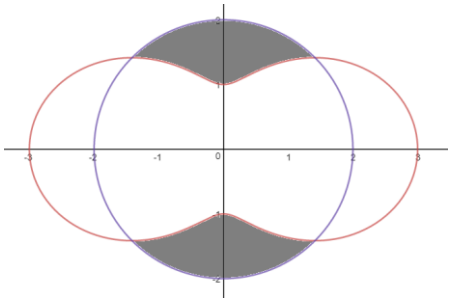
A) 6
B) 3
C) 2
D) 1/2
E) NOTA

28. If the region underneath $y = \frac{8}{x^2}$ and above the x-axis for $x \ge 1$ is divided into two regions with equal areas by the line $x = a$, then $a =$

A) 8 C) 2 E) NOTA

B) 4 D) 1

29. Find the area of the shaded region bounded by $r = 2 + \cos 2\theta$ and $r = 2$, as shown in the figure below.



A) $4 - \frac{\pi}{8}$

B) $2 + \frac{\pi}{8}$

C) $2 - \frac{\pi}{4}$

D) $4 - \frac{\pi}{4}$

E) NOTA

30. Find the volume of the solid obtained by revolving the curve $y = xe^{-x}, 1 \leq x < \infty$, around the x-axis.

A) $-\frac{5\pi}{4e^2}$

B) $\frac{5\pi}{4e^2}$

C) $\frac{\pi}{2e^2}$

D) $\frac{3\pi}{2e}$

E) NOTA